Summer Assignment
AP Calculus AB/BC

Directions: Complete each problem on a separate page. Show all work. Each problem should be completed ***without your calculator***. Be ready to hand in your work the first day of school.

Lines and Their Graphs

1. Write the equation of the following lines in point-slope form.

a. The line through the points (2, 4) and (4, -5).

b. The line with slope 3 passing through the point (4, -2).

c. The line perpendicular to 2x – 4y = 8 passing through the point (1, -2).

d. The line that contains the points (1, –5) and (–2, 4).

2. Find the equation of the straight line that passes through the point (2, 4) and is parallel to the line $2x + 3y - 8 = 0$.

3. Find the equation of the line that is perpendicular to the line 2x + 3y – 8 = 0 at the point (1,2)

4. The line with the slope 5 that passes through the point (-1,3) intersects the x axis at a point. What are the coordinates of this point?

5. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2,4) intersects the y axis?

6. Let f be a linear function such that f(2) = 5 and f(6) = –1. Find an equation for f(x).

Algebraic Manipulation

1. Simplify the following expressions.

a. $\frac{x^3}{x^{-5}}$

b. $\frac{2x^3}{y^{-5}} \cdot \frac{y^2}{3x^7}$

c. $\frac{x^2-4x-5}{x^2+2x+1}$

d. $\frac{x-4}{4-x}$

e. $(x-1)^3$

f. $x^{\frac{1}{3}}x^{\frac{3}{5}}$

g. $\frac{3x+9}{6x}$

h. $\frac{x^2}{x^{1/2}}$

i. $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

j. $\frac{1}{x+1} - \frac{1}{x-1} - \frac{2}{x^2-1}$

k. $\frac{x(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$

l. $\frac{\frac{a}{b}-\frac{b}{a}}{a-b}$

m. $\frac{2(x+h)^2+1-(2x^2+1)}{h}$

2. Solve the following for all real values of x.

a. $\frac{2}{x+1} = \frac{x-2}{2}$ b. $x^2 - 9x + 9 = 0$ c. $\frac{1}{x} + x = 4$

d. $\frac{5}{e^x+1} = 1$ e. $\sqrt{x-1} - \frac{5}{\sqrt{x-1}} = 0$ f. $2x^2 + x - 3 = 0$

g. $x^4 - 4x^2 + 2 = 0$ h. $\left(\frac{x}{2}\right)^3 = 125$ j. $2x^2 - x = 2 - \frac{1}{x}$

k. $2\sqrt{x} = x - 3$ l. $2x^2 + 2x + 1 = 0$. m. $(x-2)(x+2)(x-1)^2 = 0$

n. $\frac{(x-5)(x+3)}{(x-1)(x+1)} < 0$

3. Factor as indicated.

a. $3x^4 + 4x^3 - x^2 = x^2(\quad)$

b. $\frac{1}{2x^2+4x} = \frac{1}{2x}(\quad)$

c. $\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}(\quad)$

d. $(2x+1)^{3/2}x^{1/2} + (2x+1)^{5/2}x^{-1/2} = (2x+1)^{3/2}x^{-1/2}(\quad)$

4. Factor completely.

a. $2x^2 + 5x - 3$ b. $e^{2x} + 2 + e^{-2x}$ c. $x^3 + 4x^2 - 2x - 8$

d. $4x^4 + 3x^2 - 1$ e. $9x^4 - 25$ f. $2x^2 + 5x - 3$

5. Let k(x) = 3x + 2. Find k(a), k(2a), and k(a + 1).

6. Solve $x = y^3 - 4$ for y in terms of x.

7. Solve the system: $\begin{cases} y^2 = 1 - x^2 \\ y^2 = x^2 - 3x + 2 \end{cases}$

8. Given $f(x) = |x - 3|$ find f(1) – f(5).

9. Given $f(x) = x^2 - 3x + 4$ find f(x+2) – f(2).

10. Give $f(x) = \frac{1}{x}$ find $\frac{f(x+h)-f(x)}{h}$

11. Given f(x) = x – 3 and $g(x) = \sqrt{x}$ complete the following.

a. f(g(x)) = b. g(f(x)) = c. f(f(x)) =

12. Given $f(x) = \frac{1}{x-5}$ and $g(x) = x^2 - 5$ complete the following.

a. f(g(7)) = b. g(f(v)) = c. g(g(x)) =

13. If f(x) = $x^2 - 4x + 6$, find f(0), f(2), f(-2), f(a), f(-a), f(x + 1), f(2x), and 2f(x) – 2.

14. If $f(x) = 4 - \sqrt{3x - 6}$, find f(5), f(9), f(a + 2), f(-x), f(x^2), and [f(x)]^2.

Domain and Range

1. For what value of x is the function $g(x) = \frac{2x+1}{x+7}$ undefined?

2. Find the domain of the function.

a. $f(x) = \sqrt{9 - x^2}$

b. $g(x) = -\sqrt{x - 3}$

c. $h(x) = \frac{1}{4x^2 - 21x - 18}$

d. $k(x) = \sqrt{x^2 - 5x - 14}$

e. f(x) = $t^2 - 2t + 5$

f. g(x) = 7x + 15

g. $h(x) = \frac{2x+1}{2x-1}$

h. $k(x) = 3x - \frac{2}{\sqrt{x+1}}$

i. $f(x) = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2}$

j. $g(x) = \frac{2x^2+5x+3}{2x^2-5x-3}$

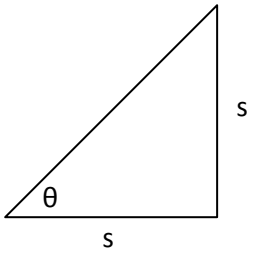
k. $h(x) = \sqrt{4x - 1} + \sqrt{x^2 - 1}$

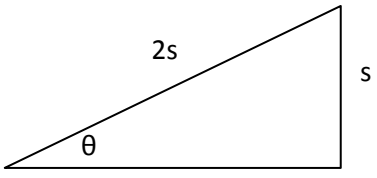
Trigonometric Functions

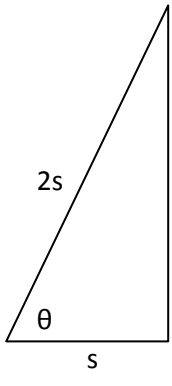
1. Solve the following for the indicated variable.

a. $3\cos x - 1 = 2$ b. $2\sin(2x) - \sqrt{3} = 0$ c. $\tan^2 x - 1 = 0$

2. Evaluate all six trigonometric functions for each θ.







3. Solve for θ from $0 \leq \theta \leq 2\pi$. Leave all answers in terms of radians.

a. $\sin\theta = -\frac{1}{2}$ b.$\cos\theta = \frac{\sqrt{2}}{2}$ c. $\tan\theta = -1$

d. $\csc\theta = \frac{2\sqrt{3}}{3}$ e. $\sec\theta = 1$ f. $\cot\theta = -\frac{\sqrt{3}}{3}$

g. $\sin\theta = 0$ h. $\cos\theta = \frac{\sqrt{3}}{2}$ i. $\tan\theta = \sqrt{3}$

j. $\csc\theta = 2$ k. $\sec\theta = -\frac{\sqrt{2}}{2}$ l. $\cot\theta = und$

m. $\sin\theta = -\frac{\sqrt{2}}{2}$ n. $\cos\theta = 0$ o. $\tan\theta = 0$

4. Evaluate each trigonometric function.

a. $\sin\frac{3\pi}{4}$ b. $\sec\left(-\frac{7\pi}{3}\right)$ c. $\cot\pi$

d. $\cos\left(-\frac{3\pi}{2}\right)$ e. $\tan\frac{11\pi}{6}$ f. $\csc\frac{2\pi}{3}$

5. Factor the expression.

a. $\sin x + \tan x = \sin x\,(\quad)$

b. $5\cos^2 x - 5\sin^2 x + \cos x + \sin x$

c. $1 - \sin^2 x$

d. $\cos^2 x + 4\cos x + 4 - \tan^2 x$

6. Simplify $\frac{\mathbf{1-(\sin x+\cos x)^2}}{\mathbf{2\sin x}}$

7. Solve $\cos^2 x + 3\cos x + 2 = 0$

8. Find the exact value of $\sin\left(\cos^{-1}\left(-\frac{1}{3}\right)\right)$

9. Find the solution of the equations for $0 \leq \theta \leq 2\pi$.

a. $2\sin^2\theta = 1 - \sin\theta$ e. $2\tan\theta - \sec^2\theta = 0$ f. $\sin 2\theta + \sin\theta = 0$

10. Which of the following expressions are identical?

a. $\cos^2 x$ b. $(\cos x)^2$ c. $\cos x^2$

11. Which of the following expressions are identical?

a. $(\sin x)^{-1}$ b. arcsinx c. $\sin x^{-1}$

Exponents and Logarithms

1. Write the equations in logarithmic form.

a. $2^6 = 64$ b. $49^{\frac{1}{2}} = \frac{1}{7}$ c. $10^x = 74$

2. Evaluate the following.

a. $\log_2 128$ b. $\log_8 1$ c. $10^{\log 45}$

d. $\log .000001$ e. $\ln e^6$ f. $\log_4 8$

g. $\log_3 \frac{1}{27}$ h. $2^{\log_2 13}$ i. $\log_5 \sqrt{5}$

j. $e^{2 \ln 7}$ k. $\log 15 + \log 4$ l. $\log_3 \sqrt{243}$

m. $\log_2 16^{23}$ n. $\log_2 250 - \log_5 2$ o. $\log_8 6 - \log_8 3 + \log_8 2$

3. Expand the logarithmic expressions.

a. $\log_2\left(x\sqrt{x^2+1}\right)$ b. $\ln \sqrt{\frac{x^2-1}{x^2+1}}$ c. $\ln\left(\frac{4x^3}{y^2(x-1)^2}\right)$

4. Combine into a single logarithm.

a. log 6 + 4 log 2 b. log x + log (x^2y) + 3log y c. $\frac{3}{2}\log_2(x-y) - 2\log_2(x^2+y^2)$

5. Solve for x.

a. $2^x = 64$ b. $10^x = 1000$ c. $\log x = 0.72$

d. $4^x = 3$ e. $\ln x = 1.09$ f. $\ln e^3 = x$

g. $\ln e^x = 4$ h. $\ln x + \ln x = 0$ i. $e^{\ln 5} = x$

j. $\ln 1 - \ln e = x$ k. $\ln 6 + \ln x - \ln 2 = 3$ l. $\ln(x+5) = \ln(x-1) - \ln(x+1)$

m. $\log_2(1-x) = 4$ n. $2^{3x-5} = 7$ o. $5^{5-3x} = 26$

p. ln(2x – 3) = 14 q. $e^{\frac{3x}{4}} = 10$ r. $2^{1-x} = 3^{2x+5}$

s. log x + log(x + 1) = log 12 t. $\log_8(x+5) - \log_8(x-2) = 1$

6. Express y in terms of x.

a. $\log y = x + 2$ b. $\ln y = 2 \ln x$ c. $\log y = 4 \log x + 3$

Graphs

1. Sketch the graphs of $y = x^2 - 4x + 3$ and $x - 2y = -6$ on the same set of axes. Find the coordinates of each intersection point.

2. Sketch the graph of each function.

a. $f(x) = \begin{cases} 1, & x \leq 0 \\ -1, & x > 0 \end{cases}$

b. $f(x) = \begin{cases} 2x, & (-\infty, -1) \\ 2x^2, & [-1, 2) \\ -x + 3, & (2, \infty) \end{cases}$

3. If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of f(x).

a. f(x) - 4

b. f(x – 4)

c. –f(x + 2)

d. 5f(x) + 3

e. f(2x)

f. $|f(x)|$

4. Sketch the graph of the following functions

a. $(x) = 1 - 2x$

b. $f(x) = \frac{1}{3}(x - 5), 2 \leq x \leq 8$

c. $f(t) = 1 - \frac{1}{2}t^2$

d. $g(t) = t^2 - 2t$

e. $f(x) = x^2 - 6x + 6$

f. $f(x) = 3 - 8x - 2x^2$

g. $g(x) = 1 - \sqrt{x}$

h. $g(x) = -|x|$

i. $f(x) = \begin{cases} x + 6, & x < -2 \\ x^2, & x \geq -2 \end{cases}$

j. $f(x) = \begin{cases} x^2, & 0 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$

5. Sketch the graph of the indicated translations of $f(x) = x^2$ on the same axis. Use a different color for each equation.

a. f(x) + 2

b. f(x+2)

c. –f(x)

d. f(-x)

e. 0.5f(x)

f. 2f(x)

6. Sketch the graph of the indicated translations of $f(x) = \frac{1}{x}$ on the same axis. Use a different color for each equation.

a. f(x) + 2

b. f(x+2)

c. –f(x)

7. Sketch the graph of the indicated translations of $f(x) = \sqrt{x}$ on the same axis. Use a different color for each equation.

a. f(x) + 2 b. f(x+2) c. –f(x)

d. f(-x) e. 0.5f(x) f. 2f(x)

8. Sketch the graph of the indicated translations of $f(x) = e^x$ on the same axis. Use a different color for each equation.

a. f(x) + 2 b. f(x+2) c. –f(x)

9. Sketch the graph of the indicated translations of $f(x) = \ln x$ on the same axis. Use a different color for each equation.

a. f(x) + 2 b. f(x+2) c. –f(x)

10. Graph the following translations of $f(x) = \sin x$ for at least two periods.

a. $f(x) = -\sin x$

b. $f(x) = \sin \pi x$

c. $f(x) = \sin(x - \pi)$

d. $f(x) = \sin(x + \pi)$

e. $f(x) = \sin x + 2$

f. $f(x) = -\sin x + 2$

11. Graph the following translations of $f(x) = \cos x$ for at least two periods.

a. $f(x) = -\cos x$

b. $f(x) = \cos \pi x$

c. $f(x) = \cos(x - \pi)$

d. $f(x) = \cos(x + \pi)$

e. $f(x) = \cos x + 2$

f. $f(x) = -\cos x + 2$

12. Graph the following translations of $f(x) = \tan x$ for at least two periods.

a. $f(x) = -\tan x$

b. $f(x) = \tan \frac{\pi}{2} x$

c. $f(x) = \tan\left(x - \frac{\pi}{2}\right)$

d. $f(x) = \tan\left(x + \frac{\pi}{2}\right)$

e. $f(x) = \tan x + 2$

f. $f(x) = -\tan x + 2$

Geometry

1. The sides of a rectangle are *x* and 3 – 2*x*. Express the rectangle's area as a function of *x*. Express the rectangle's perimeter as a function of *x*. Explain why *x* cannot equal 2.

2. The height and the diameter of a cylinder are equal. Express the volume of the cylinder as a function of its radius.

3. Give the dimensions of three different rectangles with area 6 cm^2.

4. Each leg of an isosceles triangle is twice as long as its base. Express the perimeter of the triangle in terms of the length b of the base.

5. Sketch the graph of the circle $x^2 + (y - 2)^2 = 25$. Find the circumference and the area of the circle.

6. Find the surface area of a box of height *h* whose base dimensions are *p* and *q*, and that satisfies the following conditions:

a. The box is closed.

b. The box has an open top.

c. The box has an open top and a square base with side length *p*.

7. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x.

8. A car travels 360 miles in a period of 180 minutes. Find the average velocity of the car in miles per hour over this time period.

9. A 20 foot ladder rests against a building 15 feet from the floor. How far does the ladder extend from the base of the wall? What angle does the ladder make with the ground?